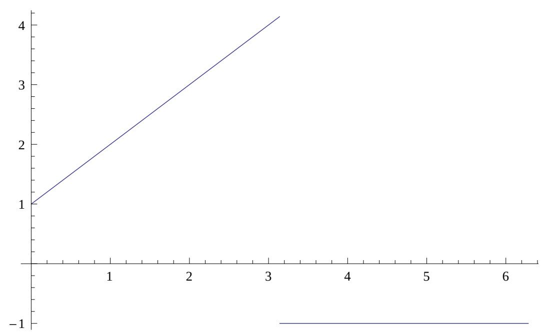
1 Math 1 HW #2

1. Let $f(x)$ be the 2π-periodic extension of the following function

$$f(x) = \begin{cases} x+1 & \text{if } 0 \le x < \pi \\ -1 & \text{if } \pi \le x < 2\pi \end{cases}$$

   (a) As $k \to \infty$ there is some exponent r such that $c_k(f) \sim \text{Constant} \times k^r$. What is the value of r?

   (b) List all the x-values near which the Fourier series for f fails to converge uniformly. At each of these x-values, calculate the overshoot of the Fourier series.



2. Recall that the 2π-periodic square wave extending

$$g(x) = \begin{cases} 1 & \text{if } 0 \le x < \pi \\ -1 & \text{if } \pi \le x < 2\pi \end{cases}$$

has Fourier coefficients given by

$$b_k(g) = \begin{cases} \frac{4}{k\pi} & \text{if } k \text{ is odd} \\ 0 & \text{if } k \text{ is even} \end{cases}$$
$$a_k(g) = 0$$

Apply Parseval's identity to the function g to get an exact formula for

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2}.$$

3. The heat equation

$$u_t = \alpha u_{xx}$$

models the flow of heat in a rod. The number $\alpha > 0$ is a constant; for this problem, suppose that $\alpha = 1m^2/s$. For any number k, the following are solutions to the heat equation:

$$\sin(kx)e^{-\alpha k^2 t}, \qquad \cos(kx)e^{-\alpha k^2 t}$$

The function $u(x,t)$ is the temperature of the rod at position x and time t. Suppose the rod has length 1 meter, and initial temperature given by

$$u(0,t) = \begin{cases} 100^\circ C & \text{if } x < .5 \text{ m} \\ 0^\circ & \text{if } x \ge .5\text{m} \end{cases}.$$

Suppose that the left endpoint of the rod is kept at a constant temperature of $100^\circ C$ and the right endpoint is kept at a constant temperature of $0^\circ C$. Give a Fourier series solution for $u(x,t)$.